Eur. Phys. J. A **25**, s01, 609–610 (2005)
DOI: 10.1140/epjad/i2005-06-020-6

EPJ A direct
electronic only

Model of binding alpha-particles and applications to superheavy elements

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Received: 21 October 2004 /
Published online: 20 April 2005 –

Abstract. The model of nuclear matter built from alpha-particles is proposed. In this model nuclei possess the molecular-like structure. Analyzing the numbers of bonds one gets the formula for the binding energy of a nucleus. The structure is determined by the minimum of the total potential energy, where interaction between alpha-particles is pairwise and the pair-potential is of Lennard-Jones type. The calculated binding energies show a good agreement with experiment. Calculations predict the stability island for superheavy nuclei around $Z = 120$.

PACS. 21.10.Dr Binding energies and masses – 21.60.Gx Cluster models

1 Introduction

We assume that atomic nuclei might be considered as a tight packing of alpha-particles. The basis of the model is assumption that the binding energy can be expressed as a sum of energies coming from interactions between alpha-particles and their self-energies. As a result the nuclear binding energy can be written as

$$E_b = A_0(6N_\alpha + n_\alpha) + C, \tag{1}$$

where A_0 determines the energy of interaction, N_α is the number of alpha-particles in the nucleus, n_α is the number of bonds between alpha-particles (for definition see ref. [1]) and C is the Coulomb energy of the nucleus. Now we can rewrite this formula for alpha-alpha interaction as for potential well. In analogy with molecular physics for the alpha-particle potential we shall take the Lennard-Jones potential.

$$V_{\alpha\alpha}(r_{ij}) = V_0\left[\left(\frac{\sigma}{r_{ij}}\right)^{12} - \left(\frac{\sigma}{r_{ij}}\right)^{6}\right], \tag{2}$$

where $V_0 = 19.2\,\mathrm{MeV}$ and $\sigma = 2.66\,\mathrm{fm}$. We can calculate positions of alpha-particles from the condition that they bring the total potential energy to its minimum. We do it by calculating the effective one-particle potential by induction, which is a good approximation. The volume of the nucleus is set on a three-dimensional grid and the first alpha-particle is positioned in the center. Then the potential is calculated in all nodes of the grid and next alpha-particle is positioned into the node with the minimum potential energy. Then we take two particles and calculate their potential and put the third in its minimum. This minimum is achieved on a circle centered in the middle between first two alpha-particles perpendicular to their connection axis. in this way on step i we have the effective potential for the particle $i + 1$ which comes from i previously located particles

$$V^i = \sum\nolimits_{j<i} V_{\alpha\alpha}(r_i - r_j).$$

In fig. 1 the separation energy of alpha-particles is plotted as a function of the number of alpha-particles in nuclei. Each peak in this picture corresponds to the nucleus with high binding energy of alpha-particles. Figure 2 shows the set of the simplest configurations. The cases (a), (c) and (d) correspond to "alpha-magic" nuclei and (b) shows that oxygen is a mixture of "classical" and "alpha-magic".

Now we can compare our values of binding energies with the experimental ones. (Experimental binding energies were taken from the website maintained by the Brookhaven National Laboratory [2].) In fig. 3 one finds the plot of the ratio of calculated to experimental binding energies depending on the number of alpha-particles. If we assume that the main trend remains when adding neutrons we obtain in the region $Z = 120$–122 and $A = 300$–310

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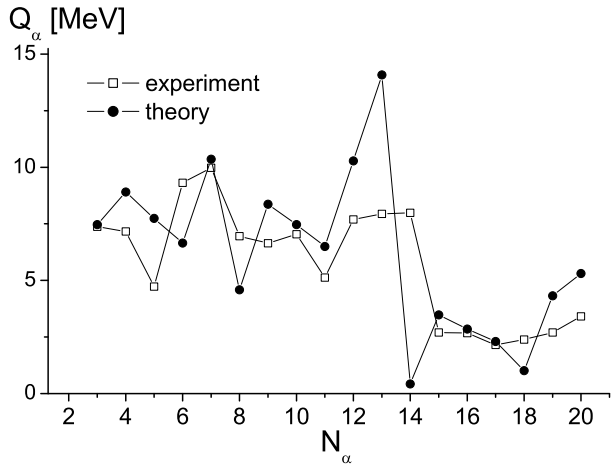

Fig. 1. Calculated one–alpha-particle separation energy (Q-value) *versus* the number of alpha-particles.

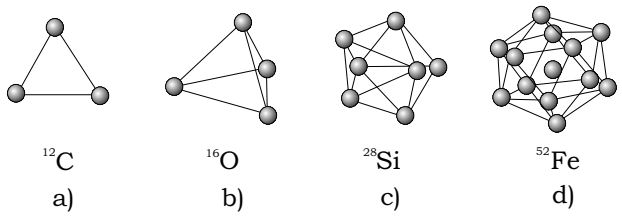

Fig. 2. The "alpha-magic" numbers. These numbers correspond to geometric figures with the maximal number of bounds per one alpha-particle.

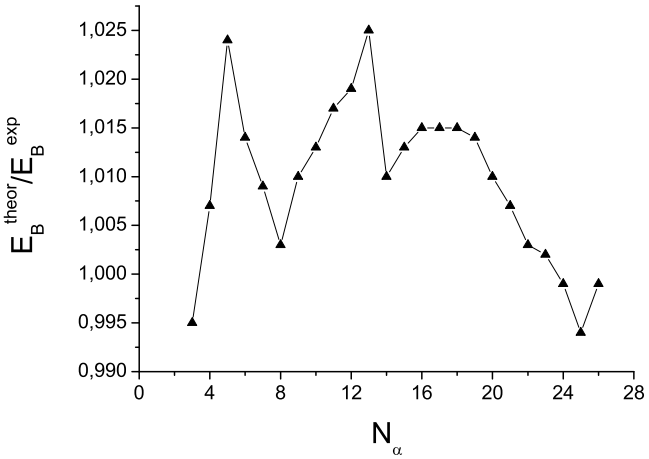

Fig. 3. The ratio of calculated to experimental binding energies. The difference is about 1–2%.

there are nuclei having high binding energy per alpha-particle and thus higher lifetime, see fig. 4. One of us, ref. [3] has suggested fullerene nuclei in this region.

The shell effects described here have a purely geometrical character. The possibility that they overlap with traditional shell effects based on Fermi-Dirac statistics and spin-orbit force is not excluded but is not considered here. This is in analogy with metal crystals, where the electron

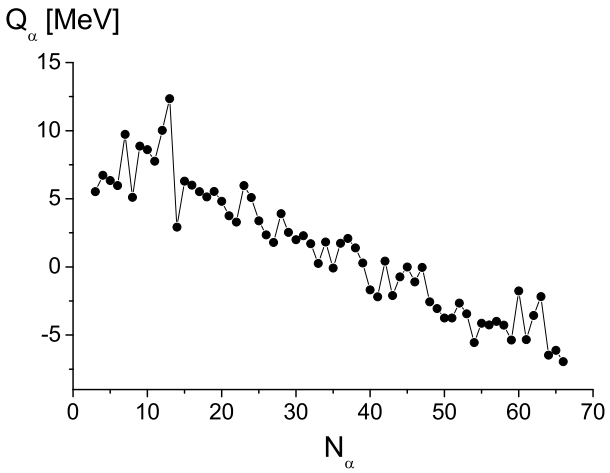

Fig. 4. One–alpha-particle separation energy *versus* the number of alpha-particles. Stability enhancement is observed at $Z = 120$ ($N_\alpha = 60$).

(fermionic) shell structure overlaps with geometric tightly packed structure of atoms. The geometric shell effects still affect the behavior of binding energy in this case. So one might expect the interplay between two effects.

2 Conclusion

We have developed a simple model of binding alpha-particles and showed that it predicts very well binding energies in a large range of nuclei, with the accuracy within the range of light and medium heavy nuclei being 1–2%, see fig. 3. For the inter-particle potential we took the one which has the form of the molecular Lennard-Jones potential. With these forces stable configurations have an approximate icosahedral symmetry. Cluster structure of the nuclear system leads to specific shell effects which do not reduce to traditional single-particle ones. Such effects could be important for stability of superheavy nuclei, where shell effects play the key role. Our preliminary calculations show enhancement of binding energy for superheavy nuclei with $Z = 120$.

This work was partially supported by Deutsche Forschungsgemeinschaft (grant 436 RUS 113/24/0-4), Russian Foundation for Basic Research (grant 03-02-04021) and the Heisenberg-Landau Program (JINR, Dubna) One of us (D.K. Gridnev) appreciates the financial support from the Humboldt Foundation.

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